# Bunching an Electron Beam by Deflection Modulation*

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*Summary*—**The coherence properties of a deflection modulated electron beam are studied for harmonic generation of electromagnetic energy in the low millimeter and possibly submillimeter wavelength region of the spectrum. A fourth-harmonic, beam-coupling experiment, at 36 Gc using a Fabry-Perot type coupler, is described. Based on these results, a proposal is presented for a beam-coupling circuit whose transverse dimensions can be made physically large compared to the free-space wavelength.**

## Introduction

ONE OF SEVERAL basic difficulties of extending classical electronic, electromagnetic sources into the submillimeter region of the spectrum is the "beam tunnel" problem. Assuming $\overline{J}$ is the beam current density, $\overline{E}$ the electric field intensity, $dA$ the element of area, and $dz$ the element of length, the well-known power expression for the interaction of an electron beam with a field is

$$P(t) = \int \overline{J}(x, y, z, t) \cdot \overline{E}(x, y, z, t) dA dz. \qquad (1)$$

In the usual situation, the spatial variation of $\overline{J}$ and $\overline{E}$ over the transverse or cross-sectional area is such that $A$ must be restricted to values which are small compared to $\lambda^2$, if the power $P$ is to have its optimum value. It is difficult to offset this reduced area problem as the wavelength $\lambda$ becomes smaller by increasing the current density $\overline{J}$, because of space-charge, beam focusing problems and cathode loading.

In this paper, a beam deflection modulation scheme is studied along with an appropriate Fabry-Perot beam coupler, in which one or both transverse dimensions of the interaction area $A$ can be made large compared to the generated wavelength, thereby achieving a quasi-optical electronics device.

While deflection modulation is one of only four basic methods of modulating an electron beam,[1] this technique has been little used in practical devices with the exception of an oscilloscope. Several proposals[2,3] for the use of deflection modulation in microwave frequency multipliers have appeared in the past, but only one experimental application has appeared recently. Kaufman and Oltman,[4] following along similar lines as Clark[3] *et al.*, reported achieving approximately two watts output from a deflection modulation system in the microwave region.

This paper further extends the previous work by showing that an extended interaction region can be used with a deflection modulation system, and by proposing a method of achieving a high interaction current by means of a multiple, two-dimensional array of electron beams.

## Deflection Modulation System

The deflection modulation system to be considered in this paper is the rectangular microwave cavity shown in Fig. 1. Assuming the cavity resonant in the dominant mode, the equation of motion associated with the $X$ direction will be taken as

$$\frac{dmv_x}{dt} = qE_x = qE_0 \cos\left(\frac{\pi y}{b}\right) \sin\left(\frac{\pi z}{l}\right) \sin(\omega t + \alpha) \qquad (2)$$

where $m$ is the electron mass; $v_x$ will be the $x$ component of velocity; $E_0$, the electric field amplitude; $b$, the transverse dimension of the cavity in the $y$ direction; $l$, the cavity length in the $z$ direction; $\omega$, the drive frequency; $t$, the time; and $\alpha$, the phase angle the electron enters the cavity.

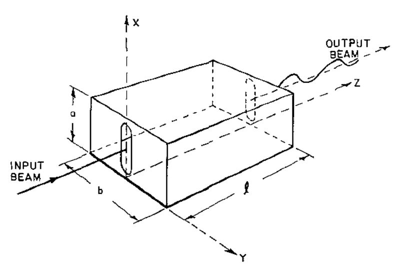


Fig. 1—$TE_{011}$ mode rectangular deflecting cavity.

Under the further conditions $y=0$, $z=vt$ and $m$ approximately constant in time, (2) can be integrated to yield

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[1] D. Gabor, "Energy conversion in electronic devices," *J. IEE*, vol. 91, pp. 128–145; September, 1944.

[2] F. K. Willenbrock and S. R. Cooke, "Modulation, Generation and Frequency Multiplication of Microwave Oscillations by a Spiral Beam of Electrons," NBC, Washington, D. C., Contract Rept. on N5-ori-76, Task order No. 1NR-078-011; June, 1949.

[3] G. L. Clark, H. von Foerster, and L. R. Bloom, "The Rotatron," Engineering Experiment Station, University of Illinois, Quarterly Rept. No. 4, Contract No. AF 19(600)-23, Urbana, Ill.; September, 1952.

[4] I. Kaufman and H. G. Oltman, "Harmonic Generation by Electron Beam Deflection," presented at Conference on Electron Devices Research, Minneapolis, Minn.; June, 27–29 1962.

$$v_x = \frac{qE_0}{2m}\left\{\left[\frac{\sin Mt}{M} - \frac{\sin Nt}{N}\right]\cos\alpha - \left[\frac{\cos Mt}{M} + \frac{\cos Nt}{N}\right]\sin\alpha + \left[\frac{1}{M} + \frac{1}{N}\right]\sin\alpha\right\} \quad (3)$$

where

$$M = \left(\frac{\pi v}{l} - \omega\right), \qquad N = \left(\frac{\pi v}{l} + \omega\right)$$

and $v$ is the initial beam velocity in the $z$ direction.

A second integration of (2) gives the displacement in the $x$ direction as

$$x = \frac{qE_0}{2m}\left\{\left[\frac{1}{M^2} - \frac{1}{N^2}\right]\cos\alpha + \left[\frac{1}{M} + \frac{1}{N}\right]t\sin\alpha - \frac{\cos(Mt-\alpha)}{M^2} + \frac{\cos(Nt+\alpha)}{N^2}\right\}. \quad (4)$$

To avoid beam divergence, it is desirable to have the transverse velocity $v_x$ equal to zero at $z=l$ when the electron exits from the cavity, irrespective of the entrance phase angle $\alpha$. This can be accomplished if the electron transit time is made equal to three half-cycles of the electric field.

Thus, evaluating $v_x$ and $x$ at time $t$ given by

$$t = l/v = 3\pi/\omega, \quad (5)$$

the result is $v_x=0$, while the transverse displacement reduces to the expression

$$x = -\frac{9\pi qE_0}{8m\omega^2}\sin\alpha. \quad (6)$$

Fig. 2 displays an instantaneous picture in time of this deflected beam in the $x$-$z$ plane after leaving the cavity. Each electron has the same longitudinal or $z$ velocity with zero transverse velocity. Hence, the entire sine wave pattern is translated in the $z$ direction with a uniform speed equal to the initial electron velocity $v$.

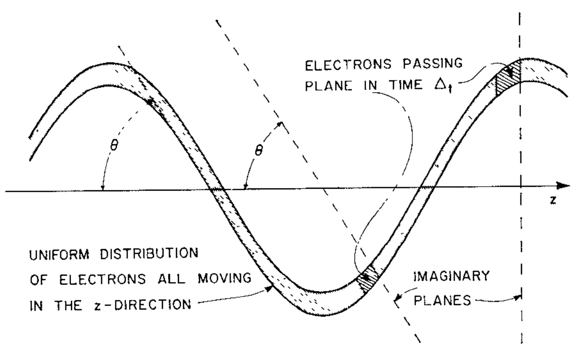


Fig. 2—Deflected electron beam having no transverse velocity.

## Current Waveform

While the charge density $\rho$ of the beam has not been changed by the deflection process, the beam has been bunched in the sense that it now can contain fundamental and harmonic current frequency components. This can readily be seen by considering the number of electrons that cross some imaginary plane as a function of time.

Referring to Fig. 2, suppose an imaginary plane is placed perpendicular to the $z$ axis. The number of electrons crossing this plane per second is constant so that the beam only has a dc current component for this situation. If, however, an imaginary plane, tilted at an angle $\theta$ with respect to the $z$ axis, is considered, the number of electrons passing this plane per second will vary in time resulting in an ac current. In particular, if the slope of the plane is made equal to the slope of the linear portion of the sine wave beam, a very nonlinear current is obtained.

Fig. 3 illustrates the schematic representation of the method used to calculate this current waveform. A rectangular, cross-section beam is assumed to be fixed in space while the imaginary plane $c$ moves in the negative $z$ direction at velocity $v$.

The lower and upper boundaries of the beam, curves $a$ and $b$, are represented by the equations

$$x_a = -\delta/2 + A\sin\left(\frac{2\pi z_a}{\beta\lambda_0}\right) \quad (7)$$

and

$$x_b = \delta/2 + A\sin\left(\frac{2\pi z_b}{\beta\lambda_0}\right), \quad (8)$$

while the tilted imaginary plane, curve $c$, is given by

$$x_c = (z_c - z_0)\tan\theta = (z_c - z_0)\frac{2\pi A}{\beta\lambda_0}\cdot \quad (9)$$

The following symbols have been used: $\beta$, the beam velocity ratio $v/c$, where $c$ is the velocity of light; $\lambda_0$, the free-space wavelength; $\delta$, the beam thickness in the $x$ direction; and $A$, the deflection amplitude.

The instantaneous current $I$ crossing the plane $c$ can be expressed

$$I = \rho v w(x_2 - x_1) \quad (10)$$

where $w$ is the beamwidth in the $y$ dimension, while $x_2$ is the $x$ coordinate of the intersection of curves $b$ and $c$, and $x_1$ is the coordinate of the intersection of curves $a$ and $c$.

Normalizing the current $I$ to the dc current $I_0$ using (9), there results

$$\frac{I}{I_0} = \left(\frac{z_2 - z_1}{\delta}\right)\tan\theta, \quad (11)$$

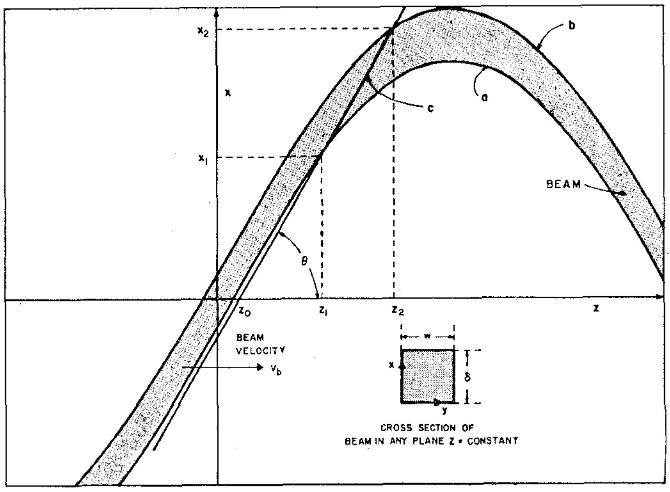


Fig. 3—Tilted imaginary plane passing through beam.

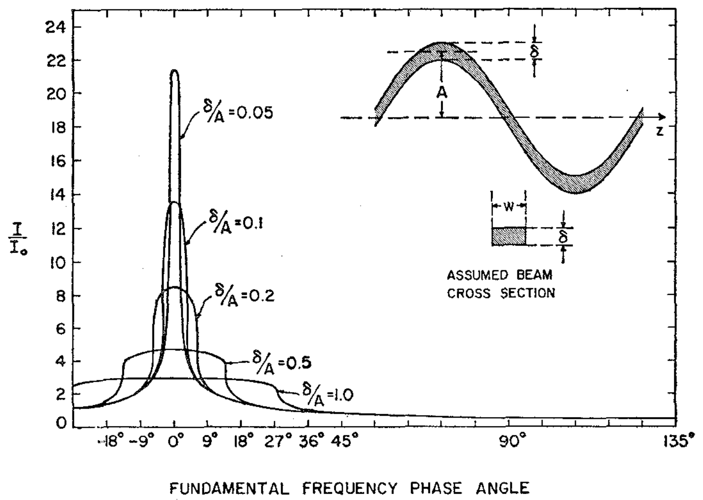


Fig. 4—Normalized current waveforms.

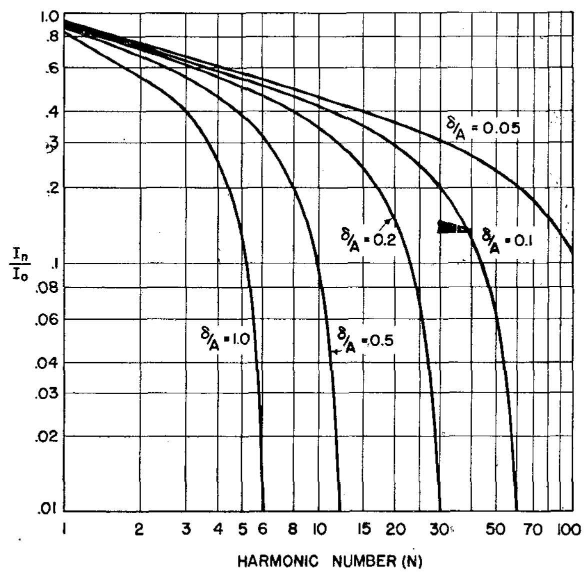


Fig. 5—Normalized harmonic current.

where $z_1$ is obtained by placing $x_a$ equal to $x_c$ in (7) and (9), while $z_2$ is obtained by placing $x_b$ equal to $x_c$ in (8) and (9).

This procedure yields two transcendental equations to be solved numerically. They are

$$kz_0 + \frac{\delta}{2A} = kz_2 - \sin kz_2 \tag{12}$$

and

$$kz_0 - \frac{\delta}{2A} = kz_1 - \sin kz_1, \tag{13}$$

where $k = 2\pi/\beta\lambda_0$.

A digital computer was used to solve these two equations. The normalized current waveforms appear in Fig. 4 and the normalized harmonic current $I_n/I_0$ in Fig. 5.

The harmonic content of this sine wave beam can be quite large for small $\delta/A$ ratios. This beam modulation scheme then places a premium on the electron gun design (small spot size) and deflection amplitude $A$ of the system to obtain high frequencies rather than the base modulating frequency $\omega$.

The smallest absolute wavelength that could be coherently generated by this type of beam would be the order of $\lambda \simeq \delta/\beta$. Thus, for a spot size $\delta$ of 0.5 mm and a 100 kv beam ($\beta = 0.55$), the minimum wavelength would be 0.91 mm.

## Fabry-Perot Coupler

There are many coupling structures that could be devised to couple to this type of deflection modulated beam. In principle, what is required is to place the axis or center line of a slow wave structure perpendicular to the tilted imaginary plane that was considered.

The structure investigated in this paper was the dielectrically loaded Fabry-Perot resonator shown in Fig. 6. For synchronism, the beam velocity $v$ and the dielectric constant $\epsilon$ of the material in the resonator must be related by the equation

$$v \cos \theta = c\sqrt{\epsilon_0/\epsilon} \tag{14}$$

where $c$ is the velocity of light.

In the case of a longitudinally bunched electron beam, Sirkis[5] has computed the power available from a Fabry-Perot system to be

$$P_n = \tfrac{1}{2} I_n^2 R_n = \tfrac{1}{2} I_n^2 \left[ \frac{L \tan^2 \theta}{8\omega\epsilon A_1 \tan \phi} \right] \tag{15}$$

where $L$ is the length of the resonator, $A_1$ the cross-sectional area, $I_n$ the driving current amplitude, $\omega$ the resonant frequency, and $\tan \phi$ the loss tangent of the dielectric.

[5] M. D. Sirkis, R. J. Strain, and W. E. Kunz, "Electron beam excitation of a Fabry-Perot interferometer," *J. Appl. Phys.*, vol. 32, pp. 2055–2056; October, 1961.

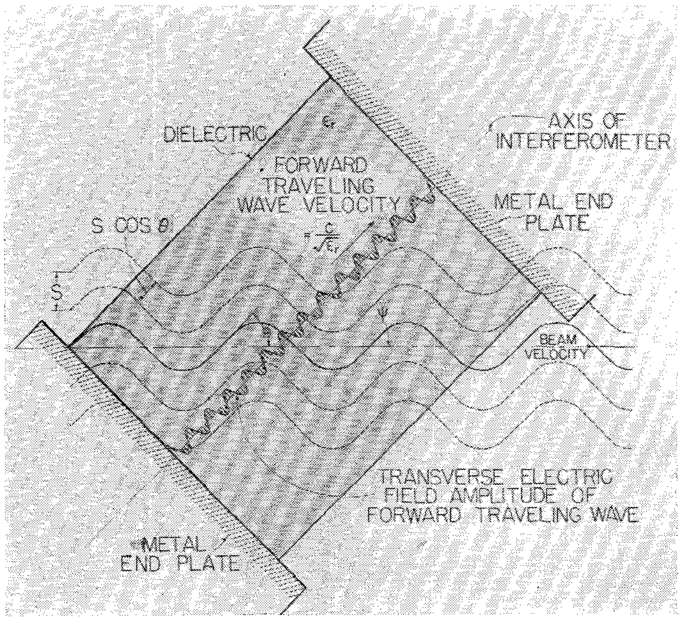


Fig. 6—Fabry-Perot beam coupler.

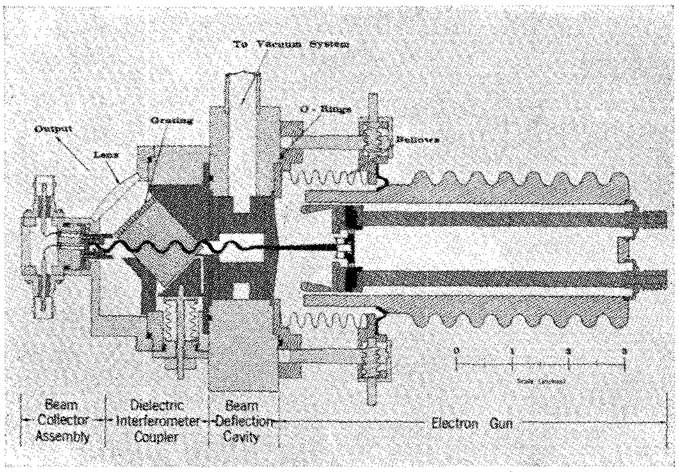


Fig. 7—Schematic of deflection modulation harmonic generator experiment.

For this experiment, a 50 kv, 0.1 a, Pierce-type gun having a spot diameter of 1 to 1.5 mm was used. A rectangular cavity, driven by approximately 30 kw of pulsed 9-Gc power was used for the deflection system. Fig. 7 gives a cross-sectional view of the experimental apparatus.

The tunable, dielectrically loaded Fabry-Perot beam-coupling structure employed Stycast[6] Hi K($\epsilon = 12\epsilon_0$) as the dielectric material. A dual collector cup assembly was used to provide for electron beam alignment and to measure the amplitude of deflection of the beam.

## Experimental Results

The frequency generated in this experiment was 36 Gc; the fourth harmonic of the beam deflecting frequency at 9 Gc. Fig. 8 displays typical oscilloscope pictures taken during the operation of this device.

[6] A product of Emerson and Cuming, Inc., Canton, Mass.

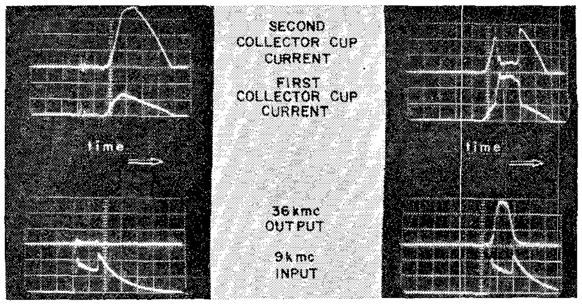


Fig. 8—Oscilloscope traces of current and power.

The four traces in the left of Fig. 8 show the second and first collector cup current, plus the 36-Gc output and 9-Gc input under conditions of no beam current, to demonstrate that the magnetron was not responsible for the harmonic power. With the electron beam on, the corresponding four traces are shown in the right hand part of Fig. 8. Approximately 0.5 mw of fourth-harmonic power were obtained.

The conversion efficiency observed was small for several reasons. First, the deflection system was not of optimum design and, hence, required appreciable drive power to achieve the desired deflecting electric field strength. Second, and more important, the beam tunnel slots in the dielectric of the Fabry-Perot resonator were made considerably larger than the beam diameter to avoid problems of dielectric charging. A calculation indicated the field strength was down by a factor of 35 over the no slot case, which would reduce the output power by over 30 db.

The authors' aims were to demonstrate the principles involved and point out how the techniques can be extended rather than optimize the output.

## Proposal for Multibeam System

Suppose that several beams, which are all located at $y=0$ but spaced a distance $s$ apart in the $x$ direction, had been injected into the deflecting cavity system of Fig. 1.

Since the deflecting electric field is uniform in the $x$ direction, all beams will be deflected in the same manner, thereby retaining their relative spacing on exiting from the cavity. These multiple beams are shown as dashed sine waves in Fig. 4 along with the original single beam.

If the separation distance $s$ were adjusted such that $s \cos\theta$ was equal to one-harmonic wavelength in the Fabry-Perot resonator, then each beam would interact in the same fashion as the original beam. Furthermore, if there were $n$ such beams used to produce the $n$th harmonic of the deflection frequency, the output power would be $n^2$ times that produced by a single beam. Also, the lowest harmonic frequency component in the composite beam would be the $n$th harmonic.

It is also conceivable that, in addition to the multiple beams stacked in the plane $y=0$, beams could be stacked at $y=\pm b$, for example, to yield a two-dimensional array of deflected beams.

Fig. 9 illustrates a possible form of the device being proposed. A nonconvergent electron gun produces a rectangular shaped beam of dimensions 0.65 by 0.65 inch. The multiple, deflected beams are obtained by drilling a two-dimensional array of holes in the anode of the electron gun, which in turn is also one end wall of the deflecting cavity.

Suppose the deflecting wavelength was 32 mm and it was desired to obtain the 16th harmonic wavelength at 2 mm. Then the anode would need to contain a 16 by 16 array of 0.016″ diameter holes spaced 0.040″ apart, for example. The $X$-band deflecting cavity would need to have a suitably large deflecting region containing a uniform electric field in both the $x$ and $y$ direction over the beam aperture.

The coupling structure suggested is a slow surface wave structure, made by cutting grooves or slots, of the approximate dimensions shown in Fig. 9, in a metal block.

To estimate the power that might be generated by this system, assume that the $z$ component of the electric field above the grating is given by the expression

$$E_z = E_0\epsilon^{-\alpha z} \sin(\gamma x + \phi) \cos(n\omega_0 t + \theta) \tag{16}$$

where $\lambda_n$ is the surface guide wavelength at the $n$th harmonic frequency, $\gamma = 2\pi/\lambda_n$, $\omega_0$ is the deflecting frequency, with $\phi$ and $\theta$ phase constants.

Ideally, the charge density distribution of the multiple beam system would be the rectangular pulse waveform shown in Fig. 10(a), but a more realistic distribution might be the sine wave waveform shown in Fig. 10(b). The current density resulting from this charge density can be expressed as follows:

$$J_z = \rho_0 v\left\{1 + \sin\frac{2\pi}{s}\left[x + A \sin\omega_0\left(t + \frac{z}{v}\right)\right]\right\}. \tag{17}$$

Using (1), the average power $P_n$ obtained at the $n$th harmonic is computed to be

$$\frac{P_n}{s^2} = \frac{1}{2}\rho_0 v E_0\left[\alpha^2 + \left(\frac{n\omega_0}{v}\right)^2\right]^{-1/2} J_n(\gamma A) \tag{18}$$

where $A$ is the peak deflection amplitude, and $J_n$ is the Bessel function of order $n$. The decay constant $\alpha$ is determined from the formula

$$\alpha = \frac{n\omega_0}{v_p}\left[1 - \left(\frac{v_p}{c}\right)^2\right]^{1/2} \tag{19}$$

where $v_p$ is the phase velocity along the slow wave surface.

To estimate the loss in the system, assume a resonant cavity is achieved by placing metal planes at each end of the slow wave circuit. The quality factor $Q$ of this cavity is given by

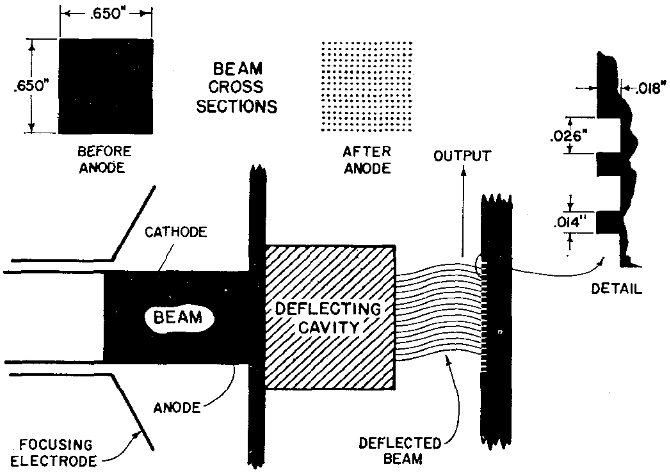


Fig. 9—Multiple beam proposal.

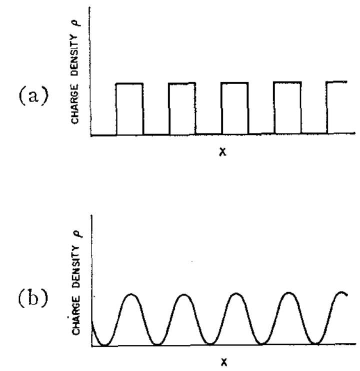


Fig. 10—Idealized and assumed charge density distribution.

$$Q = \frac{n\omega_0\xi}{2P} \simeq \frac{n\omega_0(\epsilon_0 E_0^2 s^2)}{2P\alpha} \tag{20}$$

where $\xi$ is the stored energy.

Combining (18) and (20), the final power expression is

$$P_n \simeq \frac{I_0^2 Q\alpha J_n(\gamma A)}{8n\omega_0\epsilon_0\lambda_n^2\left[\alpha^2 + \left(\frac{n\omega_0}{v}\right)^2\right]} \tag{21}$$

with $I_0$ the dc interaction current.

Eq. (20) predicts some 25 w at $\lambda_n$ equal 2 mm for the following set of parameters:

| | |
|---|---|
| beam velocity | $v=0.4c$ |
| phase velocity | $v_p=0.5c$ |
| cavity $Q$ | 20 |
| dc current | $I_0=4$ a |
| cathode load | 10 a/cm$^2$ |
| deflecting wavelength | $\lambda_0=3.2$ cm |
| deflecting power | 200 kw |
| number of beams | 256 |
| beam voltage | 50 kv |
| beam current | 25 a |

## Conclusions

A multiple-beam, deflection modulation system appears to be a rather simple method of producing a high-current, high-harmonic content beam with a large cross-sectional area. Unfortunately, the coupling structures suggested to be used with these beams have rather low interaction resistance so that the efficiency is poor. However, the quasi-optical approach using systems large compared to the wavelength is quite attractive. If better couplers could be devised, large amounts of pulsed power would be possible.

## Acknowledgment

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# Multiple Quantum Effects at Millimeter Wavelengths*

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*Summary*—**When a quantum mechanical system interacts with a radiation field it may do so by multiple as well as single quantum processes. These multiple quantum processes give rise to nonlinear effects such as harmonic generation and parametric amplification and oscillation. The density matrix formulation is used to describe these multiple quantum processes. Two- and three-level systems are considered as forms of harmonic generators and some of the desired properties of the materials to be used are described. Two methods of generating submillimeter radiation starting with optical signals are also discussed.**

## I. Introduction

Since the first work on the maser by Gordon, Zeiger and Townes [1] much interest has been focused on the possible uses of the quantum properties of matter in the field of electronics. Various types of gaseous and solid-state amplifiers and oscillators have been successfully operated in both the microwave and optical regions, and operation in the millimeter and submillimeter range seems to be possible. The basis for operation of all of these systems is the maser principle, which states simply that if the normal population distribution of an allowed transition is inverted, then there will be a net emission rather than a net absorption by the quantum system. This net emission makes possible amplification and oscillation. These processes are all essentially single quantum processes.

In addition to these familiar applications of the interaction of radiation with a quantized system, there are processes in which more than a single quantum of radiation is involved in the interaction. Such processes are called multiple quantum interactions. These processes allow various types of frequency mixing effects similar in nature to those in classically describable nonlinear elements. For this reason these quantum mechanical processes may also be called nonlinear. They are generally strong field effects and may be used for various applications such as parametric amplification and oscillation, harmonic generation, and modulation. The requirements of the quantum system for these nonlinear effects differ from those of maser applications, especially in the fact that population inversion is not necessary for most of the cases considered so far. These new requirements may make possible the use of many new materials not particularly suited for maser applications.

The millimeter and shorter wavelength range appears particularly well suited for such applications, as many atoms, molecules and crystals have strong spectra in this region.

The phenomenon of multiple quantum effects will be described in more detail and an approach which is well suited to the solution of such higher order quantum mechanical processes will be presented. This will be followed by a discussion of several possible applications.

## II. An Approach to the Problem of Multiple Quantum Transitions

For the types of problems dealt with here, the perturbation theory rate equation method does not appear as useful as some other approach which more exactly solves the equations of motion. A method of attack which is particularly well suited for such problems is the density matrix approach. A detailed discussion of the density matrix is given in [2]–[4] and its application to radiation problems in [4]–[8]. It is essentially

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